The output gap using the Hodrick-Prescott filter with a non-constant smoothing parameter: an application to New Zealand

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Abstract

We modified the Hodrick-Prescott filter to allow the smoothing parameter to vary with time to reflect changes in the relative variances of aggregate demand and supply shocks. We then used these different estimates of the output gaps to obtain estimates of the Phillips curve and to forecast inflation. We find that this modification produces estimates consistent with theory and improves the forecasts of inflation.

Key words: Hodrick-Prescott, output gap, Phillips curve

The Hodrick-Prescott filter and the problem

The output gap as a measure of economic activity or "excess demand" is the difference between actual and potential output. Theoretically, the current output gap should provide information regarding future inflation. The construction of the output gap is difficult because, among many other problems, potential output is an unobserved variable. Therefore, potential output must be estimated. There are many different ways to estimate potential output. Among these methods is the Hodrick-Prescott (1997) method, a modification of which is the primary issue of this paper.[2]

The HP filter decomposes a time series (e.g., GDP) into growth and cyclical components $Y_t = Y_t^g + Y_t^c$, where Y_t is the natural log of GDP, and Y_t^g and Y_t^c are the growth and cyclical components respectively. This decomposition assumes that series being de-trended does not contain any seasonality and, because the cycle is derived residually, it does not separate out the cycle from any irregular movements. The resulting cycle is therefore measured with error. Hodrick and Prescott (1997) minimise the variance of Y_t^c subject to a penalty for variations in the second difference of the growth term. Their filter is given by:

$$\min_{Y^g_t} = \sum_{t=1}^{T}(Y_t - Y^g_t)^2 + \lambda \sum_{t=1}^{T}[(Y^g_{t+1} - Y^g_t) - (Y^g_t - Y^g_{t-1})]^2 \qquad (1)$$

The parameter λ controls the smoothness of Y_t^g. The minimisation of (1) provides a mapping from Y_t to Y_t^g with Y_t^c determined residually. The estimate of potential output using the HP filter depends on the choice of λ, the smoothing parameter. A λ of zero corresponds to an extreme

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2 The Hodrick-Prescott paper has been cited more than 100 times in the literature (Social Science Citation Index). Four considerations are taken into account in the construction of the HP filter. First, the trend approximates the curve that business cycle researchers would draw through a time plot of the series. Second, the trend is a linear transformation of the original series. Third, lengthening the sample should not significantly change the trend, except at the sample ends. Finally, the de-trending method should be well defined, and easily reproducible. However, the HP filter is not the only de-trending technique available. See Lucas (1972), Beveridge and Nelson(1981), Blanchard and Quah (1989), Baxter and King (1994), and Kuttner (1994) for other methods. The literature contains a number of papers examining the proprieties of the HP filter, including King and Rebelo (1993), Harvey and Jaeger (1993), Cogley and Nason (1995), and Nelson and Kang (1981).

real business cycle model where all of the fluctuations in real output are caused by technology shocks because the HP trend would be the same as the series being de-trended. Conversely, as λ tends to infinity the HP trend tends toward a deterministic time trend. Following Hodrick and Prescott (1997), researchers typically set λ to 1600 for use with quarterly data, but test the robustness of their results with different values.

Kydland and Prescott (1990) argue that an economy's rate of technological change is related to current and future policy arrangements, and institutions in society. Therefore, changes, and *expected* changes, in institutions may alter the nature, magnitude, and volatility of economic shocks. Harvey and Jaeger (1993) interpret λ as a function of the ratio of the variances of aggregate demand and supply shocks. Assuming a constant value of λ in the HP filter implicitly assumes that the relative variances of demand and supply disturbances to output are time-invariant. If, for example, the variance of aggregate demand shocks is close to the variance of the aggregate supply shocks, the HP filter will not be able to distinguish the shocks.
In New Zealand the structure of the economy has been extensively reformed, with institutional changes occurring in labour relations, the banking industry, government enterprises, and so on. Furthermore, during the 1984/85 fiscal year all controls on foreign exchange transactions were abolished and the dollar floated. Subsequently, tariff levels and subsidies were significantly reduced or eliminated and import quotas were removed. Tariff reductions are set to continue into the future.

The objective of this paper is to modify the HP filter by allowing the smoothing parameter, λ to vary with time to reflect changes in the variances of aggregate demand and supply shocks. We construct a range of output gap measures using different *segmented paths* for λ (e.g., λ=1600 from 1970q1 to 1985q2 and λ=200 after 1985q2 to the end of the sample).[3] For each case, we

[3] Shazam code is attached.

compute the output gap. For each output gap, we estimate the Phillips curve equation. We then compute an intermediate range forecast of inflation. We find the root mean squared errors (RMSE) to be smaller in equations where the output gap is estimated using the HP filter with varying λ than those obtained from the HP filter with a constant λ. We conclude that policy makers whose objective is to control inflation should pay more attention to the changing nature of shocks, and use this information when forecasting inflation.

This paper is structured as follows. In the next section we compute the modified HP filter for New Zealand GDP and estimate an expectations-augmented Phillips curve to evaluate the explanatory powers of the output gaps. Section 3 is a summary.

2.The HP filters, the Phillips curve and forecasts

For our purposes, we can re-write the minimisation problem as:

$$\min_{Y^g_t} = \sum_{t=1}^{T}(Y_t - Y^g_t)^2 + \sum_{t=3}^{T}\lambda_t [(Y^g_{t+1} - Y^g_t) - (Y^g_t - Y^g_{t-1})]^2. \quad (2)$$

$$\lambda_t = 1600 \ \forall t \in \ (1970q1\text{-}1985q2),$$
$$\lambda_t < 1600 \ \forall t \in \ (1985q3\text{-}1998q4).$$

To minimise (2) we set the parameter λ_t equal to 1600 for the period 1970q1 to 1984q4. After 1984q4 we choose a set of arbitrary values for λ_t ranging between 200 and 1200 in increments of 100. Choosing the best output gap estimate is not straightforward. Since the basic application of the estimated output gap is to help forecast inflation, a natural way to evaluate which output gap measure is to be preferred is to evaluate how well each measure forecasts inflation. Therefore, we estimate Phillips curve equations with alternative estimates of the output gap, and compare their out-of-sample forecasts of inflation.

We estimate the Phillips curve equations of the form:

$$\Pi_t = \alpha E_t \Pi_{t+1} + \beta (Y - Y^g)_{t-1} + \mu_t , \tag{3}$$

where Π_t is the quarterly inflation rate defined as ΔP_t, where P_t is the natural logarithm of the CPIX (CPI excluding credit). The first term on the RHS is expected inflation measured by survey data, which is a *proxy* for expected inflation. The survey measures expectations of the inflation that will occur in the next quarter.[4] The second term on the RHS of equation (3) is the output gap, and μ_t is a Gaussian error term.

We use the Box-Cox method to estimate equation (3).[5] The predictive powers of the alternative output gap estimates are compared using the RMSE of out-of-sample forecast of inflation.

We estimate the models from 1983q2 to 1994q2.[6] Estimation results are reported in table (1). We report four regressions. Model 1 is the Phillips curve where the output gap is computed using the HP filter with $\lambda = 1600$ to estimate the trend. Models 2, 3 and 4 are similar but the output gaps are computed using the HP filter with non-constant λ's. These λ's are $\lambda = (1600{,}200)$, $\lambda = (1600{,}600)$, and $\lambda = (1600{,}1200)$ respectively. In model 1 (i.e., when the output gap is measured by the HP filter with $\lambda = 1600$) the parameter α is different from one.

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[4] The expected inflation data are from the National bank survey. There are four others surveys of inflation in New Zealand, but they are shorter and could not be used.

[5] All power transformation weight γ is identical for all variables. The output gap has non-positive values therefore it is not transformed. We also correct for first-order serial correlation. The estimates of the power weight and the first-order autocorrelation parameter are simultaneously estimated using the Savin-White method in Shazam. The divisor of N instead of N-K is used when the variance-covariance matrix is estimated. The regression includes a constant term although the Phillips curve does not include one.

[6] We had to include data from the periods of high inflation to fit the curve. In the recent periods, inflation has been very low, which made the correlation with other variables such as the output gap very weak. The Phillips curve cannot be tested without including observations of long-lasting changes in the inflation rate because inflation expectations cannot be observed without errors. In the longer sample with observations of high inflation, the error in observing expectations is perhaps small relative to the variance in actual and therefore,

Similar results are obtained from model 3 and model 4. These models are inadequate in the sense that the estimated parameters are inconsistent with theory (Phelps and Friedman natural rate hypothesis). In model 2 (i.e., when the output gap is measured by HP filter with ($\lambda = 1600, 200$)) the parameter α is statistically not different from unity. This result implies no trade-off between inflation and the output gap in the long run and it is consistent with theory. The errors of all models are white noise as indicated by the DW statistics. The values of γ are close to zero, which indicates that the models are not linear.

Next, we use the Phillips curves to compute rolling forecasts of inflation. First, we use the model's estimates for the sample 1983q2 to 1994q2 to compute out-of-sample forecasts for the next three-quarters (i.e., we forecast inflation for 1994q3, 1994q4 and 1995q1). We report the RMSE for *each* model at these forecasting horizons. Second, we add the observations 1994q3, 1994q4 and 1995q1, re-estimate the models (not reported to save space), and compute the six-quarter out-of-sample forecasts (i.e., we forecast inflation for 1995q2, 1995q3, 1995q4, 1996q1, 1996q2, and 1996q3).[7] Finally, we add these observations to the sample, re-estimate all models and compute the nine-quarter out-of-sample forecasts (i.e., we forecast inflation for 1996q4 to 1998q4). The results indicate that the inflation forecast using the output gap computed with a constant λ (λ =1600) is associated with higher RMSE than those resulting from gaps computed with varying λ. Therefore, we conclude that our technique yields relatively better estimates of the output gap than those obtained from the traditional HP filter with a constant λ.[8]

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expected inflation over the sample period.

[7] Note that we re-estimate the parameters.
[8] The RMSE associated with the second model is shaded in table (1) and is the smallest.

3. Summary

This paper argues that the traditional HP filter with constant smoothing parameter can be modified to gain an improved estimate of potential output. Harvey and Jaeger (1993) argue that the smoothing parameter λ in the HP filter is a function of the ratio of the relative variances of demand and supply disturbances. New Zealand experienced structural changes in mid 1980s. The relative variances of supply and demand disturbances have significantly changed after 1985q2, in line with structural reforms.[9] Therefore, the assumption that the HP filter's smoothing parameter should be constant is unreasonable. Accordingly, we allow λ to change over time so that its value reflects the nature and magnitude of supply and demand shocks. Determining the exact value of λ is still a problem, however.

To address this problem, values for λ smaller than 1600 for the period after 1985q2 are used and a corresponding set of output gap estimates found. These alternative output gap measures are then used to forecast inflation using Phillips curve equations. By comparing these forecasts we can infer which value of λ is more appropriate for de-trending New Zealand output after 1985q2. We find that allowing the HP filter's smoothing parameter to vary over time yields better estimates of potential output, better estimates of the output gap, and better forecasts of inflation when compared to measures obtained from the traditional HP filter with a constant λ.

For a monetary authority whose primary objective is to control inflation within a narrow band, assessments of the variance of the shocks may require a non-constant λ to be used in the estimation of potential output using the HP filter. More accurate forecasts of inflation should help improve formulation of monetary policy.

[9] Also, New Zealand floated its currency in 1985q1.

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Table (1)

The Phillips curve

$$\Pi_t^{\gamma} = \delta + \alpha E_t \Pi_{t+1}^{\gamma} + \beta (Y - Y^g)_{t-1} + \mu_t$$

Estimation covers the sample 1983q2 to 1994q2

	Model 1 $HP, \lambda(1600)$	Model 2 $HP, \lambda(1600,200)$	Model 3 $HP, \lambda(1600,600)$	Model 4 $HP, \lambda(1600,1200)$
δ^*	-0.25	-0.38	-0.32	-0.27
	(0.0001)	(0.0001)	(0.007)	(0.025)
$\alpha^{\#}$	0.76	0.94	0.85	0.79
	(0.1133)	(0.7038)	(0.3098)	(0.1530)
β^*	0.20	0.25	0.23	0.21
	(0.0001)	(0.0001)	(0.0001)	(0.0001)
γ	0.03	0.06	0.04	0.03
ρ	0.23	0.25	0.24	0.23
R^2	0.75	0.76	0.76	0.75
LLF	-31.52	-31.23	-31.29	-31.42
DW	2.02	2.01	2.02	2.02
RMSE(3)	0.2550	0.2386	0.2256	0.2435
RMSE(6)	0.2608	0.1443	0.1826	0.2399
RMSE(9)	0.2261	0.2112	0.2183	0.2240

Model 1 corresponds to the output gap measured by the HP filter with λ =1600. Models 2 3 and 4 correspond to the output gaps measured by the HP filter with λ =(1600,200), (1600,600) and (1600,1200) respectively.

ρ Is the first-order autocorrelation parameter.

γ Is the transformation parameter that maximises the log of the likelihood function.

LLF is the log of the likelihood function.

RMSE is the root mean squared error. The numbers in parentheses are the forecast horizons.

* Denotes the P-value of the t statistic for testing $H0:\delta = 0$ and $H0:\beta = 0$.

\# Denotes the P-value of the Wald statistic for testing $H0:\alpha = 1$.