Research Note N94/111

Inflation: Is It a Unit Root or a Long-Memory Process?

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1. Introduction

There has been a considerable debate within the department (especially Research Section) about the data-generating-process of the inflation rate. The issue is relevant to the estimation, forecasts, and policy making. Some believe that inflation is a unit root time series. Thus, shocks to inflation are permanent. One underlying theoretical argument is probably the Expectation-Augmented Phillips Curve, where expectation keep shifting the phillips curve to higher (lower) permanent levels as a results of an expansionary (contractionary) monetary policy. The story is believable, to some extent, but implies that the monetary authority must introduce surprises to influence real variables such as unemployment or output and economic agents are always fooled. Another story, is the case of hyperinflation. In general, the behaviour of inflation should be evolving around the political economy and particularly around monetary policy. A closer look at the inflation behaviour during the period 1925-1994 depicted in figure (1) reveals that. The mean of inflation is quite different at the beginning of the sample under the gold standard and the fixed exchange rate regimes than at the end of the sample under the float and the PTA. In the case of the expectation theory, the PTA 0-2% target policy is a good example for the role of the monetary authority affecting expectations and eventually the behaviour of the inflation rate. If we are credible, expectations should be consistent with the policy and inflation should be a mean reversion process around such policy.[1]

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[1] You may think of policy as being mean-reverting.

Empirically, if a formal statistical procedure such as the Augmented Dickey-Fuller (1979, 1981, 1984) cannot reject the null hypothesis that there is a unit root then the time series is said to be a non-stationary series with a unit root.[2] Although the ADF is a reliable test compared to other tests, it has its own problems. The problems are quite relevant to our situation. There are two problems with the ADF test. First, it is biased in small samples in the sense it fails to reject (accepts) the null hypothesis of a unit root too often. Second, it cannot handle structural breaks in the data. For example, if there is a level shift in the series for some reason (e.g., a new political or policy regime or higher inflation expectations), the ADF fits a linear line through the data and ignores the information about the change in the mean.

The solution to the power problem is to implement a more powerful test. Pantula, Gonzalez, and Fuller (1994) recommend a powerful test that is based on a maximum likelihood method. Gonzalez-Farias-Dickey (1994) use a test that shows considerable improvement in power when applied to smaller samples. However, this test still ignores structural breaks in the series. The Perron-Vogelsang (1992) test is designed to use these additional information about the shift in the series to test the null of a unit root. These two tests are used to test for the non-stationarity in the inflation series. The results are quite different from those obtained from implementing the ADF test. Hence, the tests raise some doubts about the nature of the inflation process.

It is clear that our sample size is small and there are potential structural breaks. Thus, the problems mentioned above are likely to have existed and led to the conclusion that the

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[2] The same applies to the Phillips-Perron test.

inflation series is a unit root. All that leads me to think that the ADF is an inappropriate test for the order of integration of the inflation series..

I applied these two tests to different sample periods. The Gonzalez-Farias-Dickey ML and the Perron-Vogelsang tests are applied to a long sample (1925III-1994III), a medium sample (1925III-1973I), and a short sample (1977II-1994III).[3] For a comparison with ADF test, where the unit root hypothesis can only be rejected in the large sample, the Gonzalez-Farias-Dickey test rejects the null hypothesis of unit root in both the large and the medium samples but fails to reject in the very short sample. The Perron-Vogelsang test rejected the null of a unit root in the two short samples (1973I-1994III) and (1977II-1994III) with a level shift in the data. My conclusion is that, in general, the power of any of these tests diminishes as the sample size gets smaller. However, there is a strong evidence that the inflation series is not a unit root process.

This leaves us with one more possible case, namely, inflation is either a stationary or a non-stationary but a highly persistent process. To test for the case that inflation is a stationary (non-stationary) process but highly persistent, an ARFIMA (p,d,q) model is fitted, and the parameter d is estimated from the data. Evidence is found that casts further doubts on the unit root hypothesis.

In summary, there is a significant doubt that inflation is a unit root process. The relevant sample size that we use to estimate the inflation equation is small and has potential structural breaks. Ignoring these problems in the estimation leads to spurious results. However, we have a difficult situation here. Our sample size is small, structural breaks are

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[3] The long sample represents the history of inflation under different exchange rate regimes, the medium sample covers the period before the float in the G-7 countries, and the shorter sample covers the post Bretton-Woods, the liberalisation policies in New Zealand, and the PTA.

present in the data, and we have many explanatory variables with possibly different orders of integration. To properly estimate the inflation equation over a small sample, I suggest that dummies to capture potential breaks not to be ignored, and inflation to be modeled as a fractionally integrated process.

Testing for unit roots using the Augmented-Dickey-Fuller test (1979, 1981, 1984), the Gonzalez-Farias-Dickey (1994), and the Perron-Vogelsang (1992) are presented in section 2. Theoretical ARFIMA (Autoregressive Fractionally Integrated Moving Average) model is presented in section 3. Estimation and results are found in Section 4. Section 5 is a summary.

2. Testing for a unit root

The estimates of the autocorrelation coefficient • reported in table 1 raise the first doubt on the existence of a unit root in the inflation series. These estimates are at lag 1 , and are 0.63363, 0.68992, and 0.67138 for the sample periods 1925III-1994III, 1925III-1973I, and 1977II-1994III respectively. These estimates are significantly different from 1.

The ADF and the Gonzalez-Farias-Dickey statistics are reported in table 2. Clearly, we can reject the null hypothesis of a unit root using the sample 1925III-1994III using both tests. The ADF cannot reject the null hypothesis of unit roots in the medium size sample (1925III-1973I), while the Gonzalez-Farias-Dickey test rejects the same hypothesis in both samples. Thus, inflation is a stationary series over the medium sample too. The power of the Gonzalez-Farias-Dickey test diminishes as the sample size gets smaller, both tests failed to reject the null hypothesis of a unit root in the short sample 1977II-1994III.[4]

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[4] The test fails to reject the null very often.

To account for the structural breaks in the data, the Perron-Vogelsang (1992) is applied to two shorter samples (1973I-1994III and 1977II-1994III) only. The test consists of a two-stage OLS regressions:

$$X_t = \mu_1 + \beta_1 Time + (\mu_2 - \mu_1) D_1 + (\beta_2 - \beta_1) D_2 + u_t, \qquad 1$$

where D_1 is a dummy variable that takes the value of zero before the breaking point and a value of 1 elsewhere. D_2 is a dummy variable that is equal to D_1 when D_1 is zero, and equal to the time trend elsewhere. The series X_t is the inflation series. The second regression is given by:

$$\hat{X}_t = \alpha \hat{X}_{t-1} + \sum_{i=1}^{k} \delta_i \Delta \hat{X}_{t-i} + e_t, \qquad 2$$

where $_t$ is the de-trended inflation series. The number of lags is arbitrarily set equal to 4. The results of the test are reported in table 3. The non-stationarity hypothesis is rejected at both the 5% and the 10% levels in the sample 1977II-1994III, and the 10% level in the shorter sample 1973I-1994III.

In summary, the statistics above cast serious doubt about the existence of a unit root in the inflation series. These results lead us to believe that inflation maybe a series that is non-stationary highly persistent but not a unit root series. Thus, inflation is a "long-memory" time series.

In the next section, another method to evaluate the non-stationarity of the CPI inflation series is considered. The analysis is conducted in the frequency domain rather than the time domain. The attempt is to estimate the parameter d in an ARIMA (p,d,q) process. A significant value of $0 < d < 1$ implies that inflation is a fractionally integrated process.

3. ARFIMA(Autoregressive Fractionally integrated Moving Average

Consider an ARIMA (p,d,q) model that allows fractional integration. In other words, d is less than unity,

$$\Phi(L)(1-L)^d X_t = \Theta(L)e_t \ , \quad e_t \_ (0,\sigma^2_e) \tag{3}$$

where $\bullet(L) = 1-\bullet_1 L - \ldots \bullet_p L^p$, $\bullet(L)=1-\bullet_1 L - \ldots \bullet_q L^q$, and all roots outside the unit circle with the exception that d is allowed to take any real value (i.e., not necessarily 1). Recall that if d=1 then X_t is a unit root process, if d=0 then X_t is a stationary process, and if $0 < d < 1$ then X_t is stationary but exhibits a high persistence. If $0.5 < d < 1$, the X_t is a non-stationary persistent process but not a unit root. In these latter cases, the series X_t is called "long-memory," or fractionally integrated series. A long-memory time series displays a significant dependence between observations widely separated in time. So as the interval between observations increases the autocorrelation decays is very slow. If the autocorrelation of an AR(1) process is compared to that of a fractionally integrated series, it would be very clear that the autocorrelation of the AR(1) process strikes zero faster than that of fractional series.

A time series X_t is a long-memory series if its spectral density, f_X, increases without limit as the angular frequency approaches zero. The spectral density of an ARIMA process is $\bullet^{-2d}$ with d=1 as $\bullet \rightarrow 0$, while ARFIMA series $f_X(\bullet)$ is $\bullet^{-2d}$ and $0<d<1$ $\bullet, \rightarrow 0$, so d captures the behaviour of the series at low-frequency (frequency is between 0 and $\bullet$). As d is set = 1, the series is assumed to have a unit root and, hence, called non-stationary. Empirically, a time series that has a spectra that is infinite at the origin is usually first differenced to achieve stationarity. The first difference series has no power at the origin, which suggest that the series is "over differenced." Such series is likely to be a fractionally integrated series with $0<d<1$ (see Granger and Joyeux, 1980).[5]

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[5] Granger (1980, 1988) suggested that, in general, aggregation of time series induces

4. Estimating a Fractionally Integrated Model

Estimation of d where the spectral density function • is near zero can be accomplished by first differencing the inflation rate series X_t and estimate the parameter in the model:

$$(1 - L)^{\hat{d}} X_t = \Phi^{-1}(L)\ \Theta(L) e_t = u_t . \qquad \mathbf{4}$$

In the level series, d is equal to 1+. If is equal to zero then the series is a unit root process.

The spectral density of X_t is:

$$f_x(\lambda) = |1 - e^{(-i\lambda)}|^{-2\hat{d}}\ f_u(\lambda) = [2\sin(\frac{\lambda}{2})]^{-2\hat{d}}\ f_u(\lambda), \qquad \mathbf{5}$$

where f_u(•) is the spectral density of the stationary process u_t. For a sample size N, where X_t =1,..N, the harmonic ordinates of the sample is $•_j$ = 2•j/N and j=1,...N-1. Taking the natural logarithm of equation 5 and adding and subtracting ln $f_u(0)$ yields:

$$\ln[f_X(\lambda_j)] = \ln[f_u(0)] - \hat{d} \ln[4\sin^2(\frac{\lambda_j}{2})] + \ln[\frac{f_u(\lambda_j}{f_u(0)}]. \qquad \mathbf{6}$$

Since we are interested in the activities near the origin, we can drop the last term in equation 6 and letting the periodogram at ordinate j be ln[I($•_j$)] gives:

$$\ln[I(\lambda_j)] = \ln[f_u(0)] - \hat{d}[4\sin^2(\frac{\lambda_j}{2})] + \ln[\frac{I(\lambda_j)}{f_x(\lambda_j)})] \qquad \mathbf{7}$$

This equation takes the following OLS regression equation (Diebold and Rudebusch, 1989):

$$\ln[I(\lambda_j)] = \beta_0 + \beta_1 \ln[4\sin^2(\lambda_j/2)] + \varepsilon_j\ , \quad j = 1,...k, \qquad \mathbf{8}$$

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fractional integration. Thus, we should be careful with the assumption we make about the data-generating-process (DGP) of the inflation series. Failure to estimate the underlying DGP results in spurious results.

where the *negative* of $ß_1$ is the estimate of the slope that is a consistent and an asymptotically normal estimate of d if we let the number of the low-frequency ordinates to be a function of the sample size. The errors are i.i.d across the harmonic frequencies and are equal to the term $\ln[I(\bullet_j/f_X(\bullet_j)]$ in equation 7. This equation is estimated using samples, from 1925III-1994II, 1925III-1973I, and 1977II-1994III. The theoretical asymptotic variance of the regression error is $\bullet^2/6$ is not imposed.[6] The estimates of $ß_1$ are reported in table 4. For the long sample 1925III-1994III, the estimate of d is 0.66 that is very consistent with the estimate of • in the time domain analysis. It implies that the CPI inflation is fractionally integrated series and not a unit root process. For the other shorter samples, the estimates of d are 0.80 and 0.36 respectively.

5. Summary

Several tests have been conducted to test for unit root in the CPI inflation series for New Zealand. It has been shown that the Augmented-Dickey-Fuller test (1979, 1981, 1984) is inappropriate to test for the unit root hypothesis because (1) the test is biased in small samples and (2) the test does not account for structural breaks in the data. To account for the diminishing power of the test, the Gonzalez-Dickey-Fuller maximum likelihood test (1994) is used. Results confirm that the low power of the ADF in the sense that there exists some other test that rejects the null. The Perron-Vogelsang (1992) test is used to deal with the potential structural breaks in the data. The test was applied to relatively short samples from 1977II to

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[6] Geweke and Porter-Hudak (1983) argue that the theoretical variance can be imposed on the regression to increase efficiency.

1994III and from 1973I to 1994III. The hypothesis that inflation is a unit root process was rejected at the 5% level in the longer sample, and at the 10% level in the shorter sample.

Results cast doubt on the non-stationarity of the inflation series. A model of fractional integration is used to estimate an ARFIMA (p,d,q) model over the samples 1925III-1994III, 1925III-1973I, and 1977II-1994III. The estimates of d were found to be consistent with "long-memory" time series, being significantly less than unity in all samples.

Table 1.
Autocorrelation Coefficients

Estimates of • at lag 1		
1925III-1994III	1925III-1973I	1977I-1994III
0.63363	0.68992	0.67138

Table 2
ADF and GFD tests for Unit Root

The Augmented-Dickey-Fuller Statistic (OLS)			The Gonzalez-Farias-Dickey Statistic (ML)		
1925III-1994III	1925III-1973I	1977II-1994III	1925III-1994III	1925III-1973I	1977II-1994III
-3.19*	-1.95	-1.88	-3.16*	-3.24*	-1.85

The Gonzalez-Farias-Dickey 5% critical value is -2.69. Asterisk denotes significant at both the 5% and the 10% levels.

Table 3
The Perron-Vogelsang Test for Unit Root

Sample	Break	Perron-Vogelsang	•	5% critical value	10% critical value
1973I-1994III	1985:1	-4.170#	0.5	-4.24	-3.96
1977II-1994III	1985:1	-4.341*	0.4	-4.22	-3.95

• denotes the break relative to the sample size. # is significant at the 10% level.
Asterisk denotes significant at both the 5% and the 10% levels.

Table 4
Estimate of $ß_1$

Sample 1925III-1994II	Sample 1925III-1973I	Sample 1977II-1994III
-0.34	-0.20	-0.64
(-4.9)*	(-1.91)*	(-2.88)*

t-statistics in parentheses. Asterisk denotes significant at the 5% level. d=1+ and $-ß_1=d$.

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